Shear Strength of Normal and Light Weight Reinforced Concrete Deep Beams without Web Reinforcement

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Abstract
In this paper, an analytical study is conducted to evaluate the predictive accuracy of Euro code EC2 equation and eleven (11) empirical equations proposed in the literature by several researchers for predicting the shear capacity of deep reinforced concrete beams. The results indicate that for normal strength as well as high strength reinforced concrete deep beams, the Euro code EC2 predictions are overly conservative. Among the eleven (11) empirical equations, empirical equation proposed by Karim et al is identified to be superior to the other proposed equations.

Keywords: empirical equations, shear strength, deep beams, shear span to depth ratio, concrete compressive strength

INTRODUCTION
The shear capacity (strength) of deep beams can be predicted using empirical equations or the Strut-and-Tie Model Analysis as permitted in the Codes [ACI 318-08 (2008) & Euro code EC2 (2002)]. Recently, Jung-woong Park and Daniel Kuchma (2007) used the Strut-and-Tie based method for predicting the shear strength of deep $(\frac{b}{d} \leq 1)$ beams. The proposed method employed constitutive laws for cracked reinforced concrete, considered strain compatibility, and used a secant stiffness formulation and was used to calculate the capacity of 214 normal- and high-strength concrete deep beams that have been tested in laboratories. The proposed method provided more accurate estimates of capacity than the strut-and-tie provisions in either ACI code or the Canadian Code (1994). The comparison showed that the proposed method consistently predicts the strengths of deep beams with a wide range of horizontal and vertical web reinforcement ratios, concrete strengths, and shear span-to-depth ratios (a/d) well. The use of strut-tie approach although seemingly more comprehensive, is iterative in nature and from the design perspective, somewhat cumbersome.

The purpose of this study is to assess the predictive accuracy of empirical approach used in Euro Code EC2 (2002) and out of the eleven (11) empirical equations proposed in the literature for predicting the shear capacity of the reinforced concrete deep $(\frac{b}{d} \leq 1)$ beams, identify the equation with the highest degree of accuracy. For this purpose, the predictive accuracy of Euro code EC2 (2002) and the eleven (11) empirical equations proposed in the literature for predicting the shear capacity of reinforced concrete deep $(\frac{b}{d} \leq 1)$ beams without web reinforcement is evaluated using the experimental data contained in ACCESS shear database [Rafeeqi et al (2011)]. The empirical equations proposed in the literature are based on the experimental results of reinforced concrete beams without web reinforcement and predictive capability is limited by the range of variables considered in their respective experimental programs.

The results of the study indicate that for normal strength as well as high strength $(f'_{cu} \geq 6000 \text{ psi})$ reinforced concrete deep beams, the predictive accuracy of Euro code EC2 equation is overly conservative. Among the proposed empirical equations in the literature, the equation proposed by Karim et al (2000) is identified to be superior to the other proposed equations.
Empirical Equations and Influencing Parameters

Table 1 shows the list of empirical equations along with their applicability limits used in Euro code EC2 and those proposed in the literature for predicting the shear capacity of reinforced concrete deep \( \left( \frac{d}{s} \leq 1 \right) \) beams without web reinforcement. As shown in Table 1, the equations used in Euro code EC2 and one proposed by Zsutty (1971) use cubic power function \( \sqrt[3]{f_c} \) to account for the effect of the concrete strength, whereas all other equations uses square root function \( \sqrt{f_c} \) except the equations proposed by Sarkar et al (1999) and Jin-Kuen Kim et al (1996) which uses power of \( \frac{1}{6} \) and \( \frac{2}{3} \) on \( f'_c \) respectively. The aggregate size effect on shear capacity of reinforced concrete deep beams is included in the empirical equations proposed by Russo et al (2004), Bazant and Hsu (1987) and Bazant and Jin-Kuen (1984). The equation proposed by Russo et al, in addition to aggregate size effect, also considers the yield strength of longitudinal reinforcement \( f_y \). The equation proposed by Bazant and Hsu (1987) also considers the web reinforcement parameter. The experimental data used for evaluation of the predictive capability [Rafeeqi et al (2011)], did not have information on the amount of web reinforcement therefore in the evaluation of empirical equations, the effect of web reinforcement on the shear capacity is ignored. The aggregate size factor and yield strength of longitudinal reinforcement \( f_y \) was also ignored in the evaluation study, due to non availability of desired experimental data.

Evaluation of Empirical Equations

In order to evaluate the predictive accuracy of Euro code EC2 Equation and the empirical equations for predicting the shear capacity of reinforced concrete deep \( \left( \frac{d}{s} \leq 1 \right) \) beams, test results of deep beams were selected from ACCESS shear database [Rafeeqi et al.(2011)]. A statistical term, coefficient of correlation (COR) was used along with the average \( \left( \frac{\bar{V}_{pre}}{\bar{V}_{geo}} \right) \) termed as Margin of Safety to access the predictive accuracy of Euro code EC2 Equation and the empirical equations proposed in the literature. In order to study the effect of concrete compressive strength \( f'_c \) on the shear capacity of reinforced concrete deep beams, the test data was categorized into normal and high strength \( (f'_c \geq 6000 \text{ psi}) \) reinforced concrete deep beams as shown in Table 2. Also shown in the Table 2, is the number of beams used for evaluation for each case and it varies because of the limit or constraints imposed on the variables by the individual researchers for the proposed respective empirical equations.

Euro Code EC2 Equation

Euro Code EC2 (2002) uses the following equation for estimating the shear capacity

\[
\bar{V}_{pre} = \left( \frac{d}{s} \right) \left( \frac{600}{f'_c} \right) \frac{f_y}{1000} \left( 0.5 + 0.15 \sigma_{cp} \right) \]

Eq. 1

\[
\sigma_{cp} \text{ in MPa}
\]

\[
f_y \text{ in MPa}
\]

\[
f'_c \text{ in MPa}
\]

\[
\bar{V}_{pre} \text{ in kN/m}
\]

For normal strength concrete (NSC) deep beams, the average Margin of Safety \( \left( \frac{\bar{V}_{pre}}{\bar{V}_{geo}} \right) \) is 5.70 (Table 2). This indicates that Euro code EC2 (2002) equation is overly conservative for predicting the shear capacity of short beams. For NSC as well as HSC deep beams, the COR when using the Euro code EC2 (2002) equation is 0.447 (Table 2). In case of HSC beams, Euro code EC2 (2002) which uses cubic power function \( \sqrt[3]{f_c} \) gives the value of COR which is essentially the same as COR of NSC beams. This is not similar to the trend observed for reinforced concrete slender \( (a/d \geq 2.5) \) beams [Shuaib et al (2011)] and reinforced concrete short \( (2.5 \leq \frac{a}{d} \leq 1) \) beams [Shamsoon et al (2011)]. Thus it seems that, when predicting the shear capacity of reinforced concrete deep beams, the use of cubic power function \( \sqrt[3]{f_c} \) is not a major influencing factor in reflecting the effect of the concrete compressive strength \( f'_c \).

Proposed Equations - Literature

As shown in Table 2, for NSC deep beams, the empirical equations of Karim et al (2000) and Jin-Kuen et al [Equation A, B & C (1996)] give the highest value of coefficient of correlation (COR), which comes out to be 0.804 and 0.803 respectively. The lowest COR of 0.352 is for the equation proposed by Daejoong kim et al.(1999). In case of HSC deep beams, the highest value of COR of 0.904 is for the empirical equation of Karim et al. (2000)
and the lowest COR of 0.583 is for the empirical equation of Bazant and Jin-Kuen Kim (1984).

**SUMMARY AND CONCLUSIONS**

From the evaluation study of empirical equations of Euro code EC2 and those proposed in literature to predict the shear capacity of reinforced concrete deep beams, the following conclusions can be drawn:

1) For NSC as well as HSC deep beams, the predictive accuracy of Euro code is overly conservative.

2) When predicting the shear capacity of reinforced concrete deep \(\left(\frac{d}{d} \leq \frac{2}{2}\right)\) beams, the use of cubic power function \(\frac{f_{c}}{f_{c}}\) is not a major influencing factor in reflecting the effect of the concrete compressive strength \(f_{c}\).

3) For NSC as well as HSC deep beams, the proposed empirical equation of Karim et al is considered to be the superior to the other proposed empirical equations.

**REFERENCES**

ACI Committee 318, "Building Code Requirement for Reinforced Concrete (ACI 318-08),” American Concrete Institute, Detroit.


**LIST OF TABLES**

**Table 1**: Empirical equations used in Codes and those proposed equations in the literature for predicting the shear strength of concrete in reinforced concrete members.

**Table 2**: Summary of results showing the Average Margin of Safety \(\frac{\sigma}{\sigma_{k}}\) with coefficient of correlation (COR) for NSC and HSC reinforced deep beams.
Table 1: Empirical equations used in Codes and those proposed equations in the literature for predicting the shear strength of concrete in reinforced concrete beams

<table>
<thead>
<tr>
<th>No.</th>
<th>Author’s</th>
<th>*Year</th>
<th>Empirical Equation’s</th>
<th>Comments</th>
</tr>
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<tbody>
<tr>
<td>1-</td>
<td>Russo, Somma, Angeli</td>
<td>2004</td>
<td>$f_{u,v} = f \left[ 0.05^3 \frac{d}{f_d} \left( \frac{f_{c}}{f_d} \right)^{0.15} \frac{g_{k}}{f_{y,l}} \sqrt{\frac{g_{k}}{f_{y,l}}} \left( \frac{f_{c}}{f_d} \right)^{0.15} \frac{g_{k}}{f_{y,l}} \right]$ [where $f = \frac{d}{\sqrt{\sum l_{2}}}$ and $f_{y,l}$ = Yielding Strength of longitudinal reinforcement.]</td>
<td>For High Strength Concrete Beam without Shear Reinforcement</td>
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<tr>
<td>2-</td>
<td>Eurocode EC2</td>
<td>2002</td>
<td>$f_{u} = \left( \frac{d}{2f} \right) \left( \frac{1.14}{\gamma} \sqrt{100P_{1}f_{y,t}} \right) \frac{1}{b_{n}^{0.5}} + 0.15c_{w}$</td>
<td>( f_{y,t} \leq 100 \text{ MPa} ) ( h = 1 + \frac{\sqrt{2b_{n}}}{d} \leq 2 ) where ( d ) is in mm</td>
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<td>3-</td>
<td>Karem et al</td>
<td>2000</td>
<td>$f_{u} = \gamma_{0} + \sqrt{\frac{f_{c}}{f_{d}}} \gamma_{2} \left( a_{c} - a_{c,0} \right)$</td>
<td>Where ( \gamma_{0} = 2.5 \delta_{2} = 2.5 )</td>
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<td>4-</td>
<td>S. Sarkar, O. Adwan, B. Base</td>
<td>1999</td>
<td>$f_{u} = 4.18 \left( f_{c} \frac{d}{f_{d}} \right)^{0.49}$</td>
<td>For $\frac{d}{2} \leq 2$</td>
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<td>Daejoong Kim, Woo Kim, and Richard N. White</td>
<td>1999</td>
<td>$a_{n} = 0.02 \left[ 1 - \sqrt{\frac{f_{c}}{f_{d}}} \right] \left[ \left( 1 + 100f_{c}^{0.6} \left( \frac{d}{2f} \right)^{0.6} \right) \right]$ ( r = \left( \frac{d}{2f} \right)^{0.6} \rho_{w}^{-0.01} ) ( r = ) internal moment arm length index</td>
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<td>Jin-Keun Kim and Yon-Dong Park (A)</td>
<td>1996</td>
<td>$f_{u} = 15.3f_{c}^{-1/3} \frac{d}{f_{d}} \left( 0.4 + \frac{d}{f_{d}} \right) \left( 1 + 0.07 \right)$ ( 1 \leq \frac{d}{f} \leq 3 ) ( 0.5 \leq \frac{\delta}{d} \leq 0.3 ) ( \delta = 2 - \frac{\delta}{d} ) ( \delta = 2 - \frac{\delta}{d} ) ( \delta = 2 - \frac{\delta}{d} ) ( f_{c} \geq 250 \text{ mm (9.84 in)} )</td>
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<td>Jin-Keun Kim and Yon-Dong Park (B)</td>
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<td>$f_{u} = 10.5f_{c}^{-1/3} \frac{d}{f_{d}} \left( 0.4 + \frac{d}{f_{d}} \right) \left( 1 + 0.07 \right)$ ( 1 \leq \frac{d}{f} \leq 3 ) ( 0.5 \leq \frac{\delta}{d} \leq 0.3 ) ( \delta = 2 - \frac{\delta}{d} ) ( \delta = 2 - \frac{\delta}{d} ) ( f_{c} \geq 250 \text{ mm (9.84 in)} )</td>
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<td>9-</td>
<td>Bazant, Hsu-Huei Sun</td>
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<td>$f_{u} = \left[ 0.028 \left( 1 + 3 \frac{d}{f_{d}} \right) \left( 1 + 3 \frac{d}{f_{d}} \right) \left( 1 + 3 \frac{d}{f_{d}} \right) \right] \sqrt{\frac{f_{c}}{f_{d}}} \left( \frac{1}{1 + \sqrt{2b_{n}}} \right) + 0.15$ ( \text{None mentioned in paper.} )</td>
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<td>10-</td>
<td>Bazant, Jin-Kuen Kim</td>
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<td>$f_{u} = \left( \frac{0.038 \sqrt{f_{c}} + 20.69 \sqrt{f_{c}}}{1 + 0.08 \frac{d}{f_{d}}} \right) \text{ with}$ ( \alpha = \frac{0.038 \sqrt{f_{c}} + 20.69 \sqrt{f_{c}}}{1 + 0.08 \frac{d}{f_{d}}} ) ( \alpha = \frac{0.038 \sqrt{f_{c}} + 20.69 \sqrt{f_{c}}}{1 + 0.08 \frac{d}{f_{d}}} ) ( \alpha = \frac{0.038 \sqrt{f_{c}} + 20.69 \sqrt{f_{c}}}{1 + 0.08 \frac{d}{f_{d}}} ) ( \text{None mentioned in paper.} )</td>
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<td>$f_{u} = \left( \frac{2.75}{d} \right) \left[ 2.8 + (f_{c} \frac{d}{f_{d}}) \text{Max. Aggregate size} \right]$ ( \frac{d}{f} &lt; 2.5 )</td>
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*order of listing
Table 2: Summary of results showing the Average Margin of Safety (\(\frac{V_{exp}}{V_{pre}}\)) with coefficient of correlation (COR) for NSC and HSC reinforced deep beams.

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