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Hanoi: An Embedded System for Tower of Hanoi Puzzle

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Abstract
The Tower of Hanoi (TOH) is an excellent problem for robotics research and education. From an algorithmic perspective, Natural Algorithm (NA) has proven to be a successful way to deal with such complex systems. This paper provides a NA that is capable enough to solve TOH puzzle with finite number of towers and disks. In this research work an embedded system is introduced named as Hanoi which functionalities are based on NA. In practical state this Hanoi capable to provide solution for multiple towers and multiple disks problem by using minimum number of operations to transport disks from the source tower to the target tower and minimum cost path. The main portion of Hanoi is a mechanical arm which is designed to capture a disk and to move it from one tower to another tower. The arm can move freely in both clock-wise and counter-clock-wise direction. In this paper, provided solution is for six towers and five disks have been shown. Due to these huge tower-disk combinations Hanoi can be applied in industrial purpose.

Keywords: ATmega8, Natural Function, Natural Algorithm, Optical Sensor, Robotic Arm, Stepper Motor.

INTRODUCTION

Tower of Hanoi is known as a mathematical puzzle or game. It consists of three towers and multiple disks of different sizes. At the beginning all disks are placed arbitrarily in a tower named as "source tower". The objective of the puzzle is to move the entire disk's stack to destination tower named as "target tower" according to the size of the disk (i.e. the largest disk will place at the bottom position in the stack of the target tower and the rest of the disks are to be placed consistently in the descending size of the disk) with help of intermediate tower named as "rest tower". Minimum number of operations will be required to transport disks from the source tower to the target tower [Egger J et al., 2007] [Janssen G et al., 2009] [Koorland MA et al., 2005]. There has two major parts of TOH puzzle [Java Games and Puzzle: mazeworks.com/hanoi], are

- Number of movements of the disks for a particular combination of tower and disk.
- Movements of the disks between the source towers to the target tower.

When there are 3 towers in the puzzle, then the solution is quite straight forward [Educational Resources: superkids.com], to solve the TOW puzzle required minimum number of moves is $2^d - 1$, where d indicates the number of disks. But with the increase of number of towers and disks, complexity of the problem and the algorithm size increases dramatically that involves significant number of equations.

In this paper, natural algorithm has been developed that provides the solution for TOH problem with finite number of towers and disks, details algorithm related description provides in our another papers [A. S. Zaforullah Momtaz et al.,, 2011] In this research work an embedded system is introduced named as Hanoi which functionalities are systematized by NA. The main portion of Hanoi is a mechanical arm which is designed to capture a disk and to move it from one tower to another tower. At the bottom position of the arm, there is an electromagnet attached to capture the disks as the disks are made of steel. The arm can move in vertical direction with the help of a stepper motor and also can move in horizontal direction with the support of another stepper motor [Stepper motor: wikipedia.org/wiki/Stepper_motor].

To reduce the complexity of mechanical system a specific location is created for each of the tower on the periphery of a circle. And the arm is placed just at the periphery of it i.e. there is 60° angular displacement in between two towers. The arm can move freely in both clock-wise and counter-clock-wise direction.

NATURAL ALGORITHM (NA)

In the puzzle, there are two terms- tower and disk. For a specific number of towers the number of disks may vary. It may be above, or below, or equal to the number of towers. Depending on these two variables, different equations are obtained for the natural function. The core of natural algorithm is obtained from natural function 'f (t, d)'. According to this function, whole algorithm is classified into some parts for a specific number of towers and also a specific number of disks.

Let ‘t’ and ‘d’ are two variables namely towers and disks respectively. These two are defined as ($0 < t \le n$) and ($0 < d \le m$) where ‘n’ and ‘m' represents the maximum number of the towers and disks respectively. Thus we have the terms t, d & N and t, d > 0. Using these two variables, the ‘Natural’ function is implemented to find out the total number of movements. Here ‘m’ is considered as 5 and n is considered as 6. According to different number of towers and disks the number of movements is shown in below tables and for each function the covered segment is positioned by gray color. For this limitation the function is defined as,

A. $f(t, d) = 2^d - 1$ when, t = 3

For any tower-disk combination, only 2 towers are involved in each movement. When the TOH puzzle is arranged with 3 towers, movements may involve the left most 2 towers or right most 2 towers or the left most and right most tower only.

Table 1: Number Of Movements Of 6 Towers For 5 Disks (t=3)

t \ d	1	2	3	4	5	6
1	-	1	1	1	1	1
2	-	-	3	3	3	3
3	-	-	7	5	5	5
4	-	-	15	9	7	7
5	-	-	31	13	11	9

B. $f(t, d) = 2d - 1$; when, t>d

When the puzzle is arranged using of multiple towers, where the number of towers are more than the number of disks i.e. t > d, then the observation is found that each of the disk takes 2 movements except the largest disk that takes only 1 movement.

Table 2: Number Of Movements Of 6 Towers For 5 Disks (t>d)

t \ d	1	2	3	4	5	6
1	-	1	1	1	1	1
2	-	-	3	3	3	3
3	-	-	7	5	5	5
4	-	-	15	9	7	7
5	-	-	31	13	11	9

C. $f(t, d) = 4d - 2t + 1$; when, $t \le d \le (t(t-1)/2)$

In limit $t \le d \le (t(t-1))/2$ some disks of the puzzle take 4 movements, some take two and the largest disk takes only a single movement.

Table 3: Number Of Movements Of 6 Towers For 5 Disks ($t \le d \le (t(t-1)/2)$)

t \ d	1	2	3	4	5	6
1	-	1	1	1	1	1
2	-	-	3	3	3	3
3	-	-	7	5	5	5
4	-	-	15	9	7	7
5	-	-	31	13	11	9

PROPOSED METHODOLOGY

NA provides both the number of movements of the disks and the directions of movements of the disks required. Here an additional algorithm is added with NA in order to perform both the mathematical as well as mechanical operations. All steps that require computing a TOH puzzle, consist of three towers and four disks, is shown in table 4

Table 4: Steps In Details For 3 Towers And 4 Disk Puzzle

Steps	t1	t2	t3
0	d1,d2,d3,d4		
1	d2,d3,d4	**d1**	
2	d3, d4	d1	**d2**
3	d3, d4		**d1**, d2
4	d4	**d3**	d1,d d2
5	**d1**, d4	d3	d2
6	d1, d4	**d2**, d3	
7	d4	**d1**, d2, d3	
8		d1, d2, d3	**d4**
9		d2, d3	**d1**, d4
10	**d2**	d3	d1, d4
11	**d1**, d2	d3	d4
12	d1, d2		**d3**, d4
13	d2	**d1**	d3, d4
14		d1	**d2**, d3, d4
15			**d1**, d2, d3, d4

A. INPUT AND OUTPUT SYSTEM

The mechanical portion gets instruction from the core algorithm. Using the natural function ‘f (t, d)’, (where t stands for towers and d for disks) the problem is classified to solve. The whole problem is classified into some steps as equal to the number of total movements of the given tower-disk combination. NA ensures the minimum movements and minimum cost path. Table 1 or Table 2 or Table 3 contains the list of the minimum movements of different combination of towers and disks. The operation flow diagram is shown figure 1:

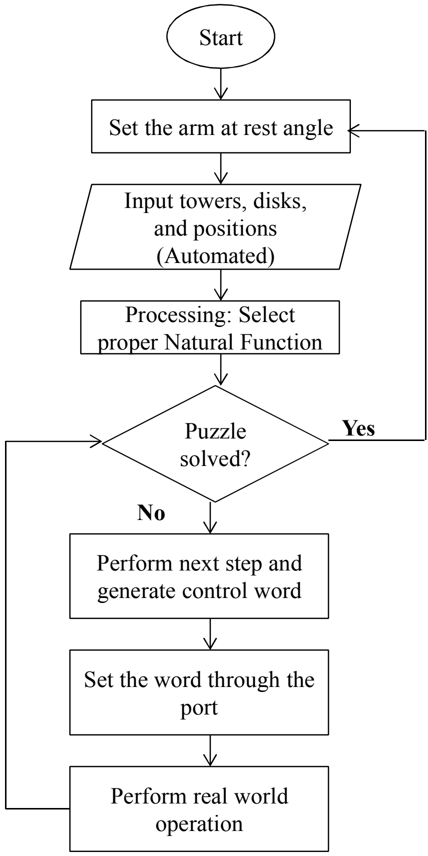

Figure 1: Hanoi Embedded System model

Proposed system provides the output in the real world by moving all the disks from source tower to destination tower using the temporary towers maintaining all the rules. Firstly, the towers are placed at the appropriate positions. Then the disks are placed at the source tower. After placing the disks a command is given that 'm' numbers of disks are placed at z^{th} tower (source tower) position. After getting all the information the base moves towards source tower by determining the shortest path which could be clockwise or anti-clockwise. Here an optical sensor is used to identify this. After getting it, the central function initializes a two dimensional array to hold the number of disks and towers of the puzzle and also classify the puzzle to solve using the appropriate function. And then send one by one disk movement instruction by setting the exact control word. Normally the ARM is placed at the top most position. When a disk is needed to be lifted from source tower it is moved in downward direction. When the top of the disk is obtained, the motion of the ARM is stopped. Then the electromagnet is energized to capture the disk. After capture the disk, the ARM is moved in the upward direction. When the top most position is obtained the ARM is moved to set it on the top of the destination tower of that disk to place it there. Here 2 sensors are used to detect the top and the bottom most position. And another sensor is used to detect the top position of the disk stack and of the surface of the base.

B. MACHINE INTERFACING AND HARDWARE OPERATION

At the early stage [A. S. Zaforullah Momtaz et al., 2011]; system is designed for Intel Pentium IV machine where communication took place through parallel port (LPT 1). At the improvisation of hardware design then we introduced ATmega8 [ATMEGA8:www.atmel.com/devices/atmega8.aspx] microcontroller with MUX [Multiplexer: wikipedia.org/wiki/Multiplexer]. According to system communication demand it requires 25 I/Os where ATmega8 is only capable to provide 23 I/Os. That implies still it has lacking of 5 I/Os. So we introduced a MUX of 16 inputs to fulfill the requirements. ATmega8 is used to control the whole system provides 14 bits information for communication purpose. The microcontroller receives only 1bit information from MUX. MUX obtained 16 bits inputs from the sensors attached at different positions where towers provide 6 bits, disks provide 6 bits, ARM and its body provides 3 bits, and tower base provides 1bit input. There are total 6 optical sensors is used to detect the towers. The sensors are arranged such a way that when a tower is placed in any of the 6 places the corresponding sensor will provide logic '1' to notify that a tower is placed at that position otherwise the sensor will provide logic '0'. The output of these sensors is used as the input of the MUX. Tower positions will be detected from the input of MUX. Again 6 optical sensors are used to detect the disk stack. The sensors are arranged such a way that when disk stack is placed in any of the 6 places the corresponding sensor will provide logic '1' to notify that the disk stack is placed at that position otherwise the sensor will provide logic '0'. The output of these sensors is used as the input of the MUX. Disk stack positions will be detected from the input of MUX. Below figure 2 represents the layout for circuit and other components of Hanoi.

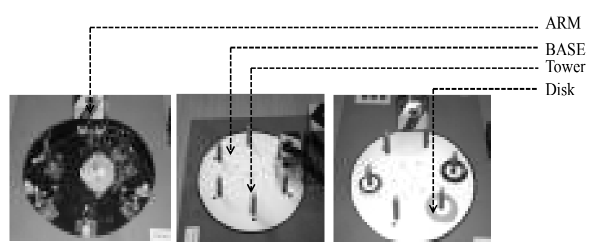

Figure 2: Layout for Circuit and other components of Hanoi

Depending on the output of the MUX the system program generates the appropriate control word and then sends it to the machine. After getting this control word it is classified according to the machine requirements. 8 bits of the word is distributed for the stepper motors. 1 bit is allocated to handle the electromagnet attached at the bottom of the ARM. Other 4 bits are distributed as the selector input of the MUX. Block diagram of hardware operation is shown in figure 3.

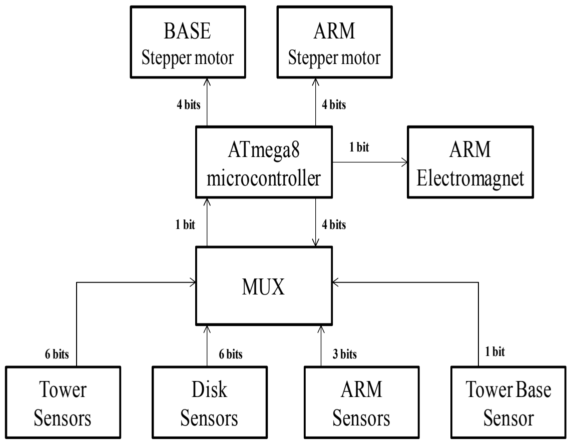

Figure 3: Block diagram of bitwise operation

RELATED WORKS

Dictionary of Mathematics that the Tower of Hanoi puzzle is of ancient Indian origin [Cicerone K et al., 2006], as the above rather widespread legend would also suggest. But it seems that it was actually invented from scratch probably together with the accompanying legend only in 1883 by the French mathematician Eduard Lucas [Math around the World: lawrencehallofscience.org/java/tower]. At least there is no known written record about the puzzle or the legend prior to 1883. He was inspired by a legend that tells of a Hindu temple where the pyramid puzzle might have been used for the mental discipline of young priests. Legend says that at the beginning of time the priests in the temple were given a stack of 64 gold disks, each one a little smaller than the one beneath it. Their assignment was to transfer the 64 disks from one of the three poles to another, with one important provisional large disk could never be placed on top of a smaller one. The priests worked very efficiently, day and night. When they finished their work, the myth said, the temple would crumble into dust and the world would vanish.

Professor Lucas called his invention "Tower of Hanoi". The singular refers to just one tower of disks to be dismantled and reassembled on a different peg. Dictionary and many other sources from the past 25 years use the plural, "Towers of Hanoi". This could reflect a growing belief in the ancient origin of the puzzle (the legend speaks about three towers - one could imagine three pyramidal pagodas with rather thin golden rings placed on their slanting roofs), or it could simply reflect the fact that the language is a living entity whose continuous evolution is impossible to stop. I have done a little survey of the sites listed below, and I found that about 26 of them respect the name given to the puzzle by its inventor and use the singular "Tower of Hanoi", whereas about 29 of them use the plural "Towers of Hanoi".
There are two main aspects involved in the TOH puzzle. These are the number of towers and the number of disks. Recently different robots are implemented to perform the puzzle. One of them used a touch screen and 3 touch pins to move the disks from source tower to the destination tower [Recursive tower of Hanoi: www.convict.lu/Jeunes/Hanoi/Tower_of_Hanoi.html]. Another robot is implemented, which have 3 towers at the 3 corners of a triangle and having an arm at the center of the triangle to move the disks. The mentioned robots are able to solve only 3 towers puzzle. But in this paper, NA can solve the TOH problem with 5 disks and 6 towers.

CONCLUSION

TOH is not just a puzzle but also a popular mathematical problem that needs special tools for its solution. That's why now-a-days TOH based robots are very popular. In this paper we have represented an efficient method to solve TOH puzzle for multiple towers and multiple disks problem by using minimum number of operations and minimum cost path.

Due to hardware limitations still we are not able to build such a system which can deal with 16 towers and 15disks, however the NA algorithm for this combination is already implemented.
There are total 6 tower positions created. Using any three of these many three towers holding 60° gap in between two towers and maximum 5 numbers of disks combinations can be made. Again there may be one tower gap combination can be possible. Due to these huge tower-disk combinations Hanoi should be a nice toy for kids. And also this mechanical system can be applied in industrial purpose.

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