Directed Graph Algorithms for Tours – A Case Study

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Abstract
In this article, we consider graph theory applications to find out the path of a real world problem. When dealing with a real world problem, the most adequate algorithm has to be chosen in order to solve the given problem in the most efficient way. We apply four algorithms separately for the problem and find out the shortest path by using these algorithms. Lastly, we compare the paths that we get by these algorithms with the solution we get by assignment problem. The main purpose of this research is to find out which algorithm is suitable out of all algorithms.

Keywords: shortest path, kruskal’s algorithm, prim’s algorithm, reverse delete algorithm, dijkstra’s algorithm, assignment method.

INTRODUCTION
Graph theory is one of the most developing branches of Mathematics. It is widely applied in subjects like Computer Technology, Communication Science, Electrical Engineering, Architecture, Physics, and Operations Research etc. In the earlier stages, it was called shim topology.


In this article, we are using four algorithms namely, Kruskal’s algorithm, Prim’s algorithm, Reverse delete algorithm and Dijkstra’s algorithm to find the shortest distance between two places.

Prim’s Algorithm
Prim’s algorithm is an algorithm in graph theory that finds a minimum spanning tree for a connected weighted graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. The algorithm was discovered in 1930 by mathematician Vojtěch Jarník and later independently by computer scientist Robert C. Prim in 1957 and rediscovered by Edsger Dijkstra in 1959. Therefore, it is sometimes called the DJP algorithm, the Jarník algorithm, or the Prim-Jarník algorithm.

Kruskal’s Algorithm
Kruskal’s algorithm is an algorithm in graph theory that finds a minimum spanning tree for a connected weighted graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. If the graph is not connected, then it finds a minimum spanning forest (a minimum spanning tree for each connected component).

Reverse-Delete Algorithm
The reverse-delete algorithm is an algorithm in graph theory used to obtain a minimum spanning tree from a given connected, edge-weighted graph. If the graph is disconnected, this algorithm will find a minimum spanning tree for each disconnected part of the graph. The set of these minimum spanning trees is called a minimum spanning forest, which consists of every vertex in the graph. This algorithm is a greedy algorithm, choosing the best choice given any situation. It is the reverse of Kruskal’s algorithm. Kruskal’s algorithm starts with an empty graph and adds edges while the Reverse-Delete algorithm starts with the original graph and deletes edges from it.

Dijkstra’s Algorithm
Dijkstra’s algorithm, conceived by Dutch computer scientist Edsger Dijkstra in 1959,[1] is a graph search algorithm that solves the single-source shortest path problem for a graph with non-negative edge path costs, outputting a shortest path tree. This algorithm is often used in routing. For a given source vertex (node) in the graph, the algorithm finds the path with lowest cost (i.e. the shortest path) between that vertex and every other vertex. It can also be used for finding costs of shortest paths from a single vertex to a single destination vertex by stopping the algorithm once the shortest path to the destination vertex has been determined.

Assignment Problem (Hungarian Method)
This method of assignment provides an efficient means of finding the optimal solution without having to make a direct comparison of every option. It operates on a principle of matrix reduction.
Mathematical Formulation of the Problem
The route map with distances is as follows:

Description of the problem: A person wants to visit some of the tourist places of Tamilnadu like Chennai, Kanchipuram, Palani, Kumbakonam, Madurai, Kanyakumari and Rameswaram. He/she has to start from Chennai, visit all the other places only once and reach Rameswaram at the end. In this process all the four algorithms are applied to find out the shortest path for this trip.
The route map with distances is as follows:The numbers near the edges indicate the distance between the stations.

Problem Solving using Algorithms and Assignment Problem
We are going to solve the problem using four algorithms.

Prim’s Algorithm:
By applying the steps of Prim’s Algorithm the final graph we get is

First we select a starting vertex. Here the starting vertex is Chennai. From Chennai the next least vertex is Kanchipuram with 71 Kms. From kanchipuram the next least vertex is kumbakonam with 250 Kms. From Kumbakonam next least vertex is Madhurai with 197 Kms. From Madhurai the next least vertex is Palani with 119 Kms. Even though from Palani the next least vertex is Madhurai, according to the problem a vertex which is visited already should not visit again, so next least vertex is Kanyakumari with 361 Kms. From Kanyakumari the last vertex is Rameswaram with 295 Kms.

Therefore the final route according to Prim’s algorithm is:
Chennai → Kanchipuram → Kumbakonam → Madhurai → Palani → Kanyakumari → Rameswaram.
The total distance = 71 + 250 + 197 + 119 + 361 + 295 = 1293 km.

Kruskal’s algorithm:
By applying the steps of Kruskal’s Algorithm the final graph we get is

The first least distance edge is from Chennai to Kanchipuram with 71 Kms. The next least distance edge is from palani to Madhurai with 119 Kms. The next least distance edge is from Madhurai to Rameswaram with 167 Kms. The next least distance edge is from Madhurai to Kumbakonam with 197 Kms. next least distance edge is from palani to Kumbakonam but it is forming cycle so next least distance edge is from Madhurai to Kanyakumari with 242 Kms. Next least distance is from Kanchipuram to Kumbakonam with 250 Kms. By applying the algorithm and considering the problem we get the final path as follows:
Therefore the final route according to Krusal’s algorithm is:

Chennai → Kanchipuram → Kumbakonam → Madurai → Palani → Kanyakumari → Rameswaram.

The total distance = 71 + 250 + 197 + 119 + 361 + 295 = 1293 km.

Reverse Delete Algorithm:
By applying the steps of Reverse Delete Algorithm the final graph we get is

The algorithm will start with the maximum distance edges in this case
Palani to Kanchipuram is the maximum distance edge with 368 Kms.
Since deleting this edge does not disconnect the graph. Proceeding in
this manner by considering the algorithm steps and description of the
problem we get the final map as follows:

Therefore the final route according to Reverse Delete algorithm is:

Chennai → Kanchipuram → Kumbakonam → Madurai → Palani → Kanyakumari → Rameswaram.

The total distance = 71 + 250 + 197 + 119 + 361 + 295 = 1293 km.

Dijkstra’s algorithm:
By applying the steps of Dijkstra’s Algorithm the final graph we get is

First of all we create distance list, visited list, previous vertex list.
Updating the tables according to algorithm steps and considering the
problem description we get the final route as follows:

Therefore the final route according to Prim’s algorithm is:

Chennai → Kanchipuram → Kumbakonam → Madurai → Palani → Kanyakumari → Rameswaram.

The total distance = 71 + 250 + 197 + 119 + 361 + 295 = 1293 km.

Applying Assignment problem:
The final route according to the assignment problem is

If the same problem is solved by Hungarian method (applying the steps of Hungarian method) the final route is as follows:
Therefore the final route according to assignment problem is Chennai → Kanchipuram → Kumbakonam → Palani → Madhurai → Kanyakumari → Rameswaram.
The total distance is 71 + 250 + 229 + 119 + 242 + 295 = 1206 Kms.

CONCLUSION
The solution clearly states that Chennai → Kanchipuram → Kumbakonam → Madhurai → Palani → Kanyakumari → Rameswaram with the total distance of 1293 kms is an optimum route map. At the same time this problem has even proved that it is not compulsory for all the directed graphs to differ in their solution since all the four algorithms gave the same optimal solution to the problem. But it differs with the solution by using assignment problem with the path as Chennai → Kanchipuram → Kumbakonam → Palani → Madhurai → Kanyakumari → Rameswaram with the total distance of 1206 kms.
Finally we can conclude that the assignment problem is giving the optimal solution compared to the above discussed algorithms.

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