**Bending Analysis of Isotropic Rectangular Plate with All Edges Clamped: Variational Symbolic Solution**

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**Abstract**

An alternative variational approach is developed for the bending analysis of thin isotropic clamped plate. The method by-passes the tedious and rigorous solution of plate’s differential equation of equilibrium involved in the classical and numerical methods. The method is a modification of Ritz variational approach. It is based on total potential energy. By the formulation, the deformation surface of the clamped plate with uniformly distributed load is approximated to be the sum of products of constructed polynomials in the x and y axes. The constructed polynomials satisfy the plate’s geometrical boundary conditions in addition to being interdependent and continuous. The sum of product of the constructed polynomial is substituted into plate’s differential equations and then solved through minimization principle. Consequently, the deflections and the bending of the thin isotropic clamped plate are obtained in analytical form thus enabling the evaluation of these quantities at any arbitrary point on the plate with uniformly distributed load at various plate aspect ratios of 1.0 to 2.0. The solution is done for first, second, third and four terms- polynomials representing 1st, 2nd, 3rd and 4th approximations respectively. The variational procedure elucidates the ease and convergence of the results.

**Keywords:** clamped rectangular plate; deflections; moments; uniform load; variational method

**INTRODUCTION**

Plates are plane two dimensional structural elements used extensively in Mechanical, Aeronautical and Civil Engineering to bear heavy loads due to its weight and economy. Ventsel and Kraumontather (2004) classified plates into thin, membrane and thick. Plates may be either isotropic or orthotropic. The modelled plate of the research is steel with the poisson ratio, \( \nu \), of 0.30. The deflection of isotropic thin rectangular plates clamped at four edges and under the action of uniformly distributed load has received considerable attention due to its technical importance (Imrak & Gerdemeli, 2006) thus, the essence of this research.

There are many classical solutions for isotropic linear elastic thin plate. Timoshenko & Woinowsky-Kreiger (1959) developed classical solutions for thin plate. Approximate solutions have also been suggested by notable researchers, but there is notably loss of accuracy (Wang & El-Sheik, 2005). Various methods have been developed in the evaluation of rectangular plates with various boundary conditions: Hencky (1913), Timoshenko (1938), Evans (1939), Young (1940), Hutchinson (1992), Wang et al (2002), Taylor & Govindgee (2004). These are generally accepted as approximate methods. Popular methods commonly used in evaluating the maximum deflections for the clamped thin plates are classical and numerical methods (Finite Element Method- FEM, Finite Difference Method-FDM, and Finite Strip Method-FSM). For the classical solution by Timoshenko (1959), two main approaches have been developed for obtaining the solution of maximum deflection for clamped thin rectangular plates under uniform load. These are double cosine series (Szilard,1974) and the superposition method as a generalization of Hencky’s solution. Hencky's solution is well known to have accelerated convergence but possess risk with regard to overflow/underflow problem in evaluation of trigonometric functions. Hutchinson (1992) extended the works of Timoshenko (1959) and evaluated deflections for the uniformly loaded rectangular plate. However, determination of the numerical values of maximum deflections for a rectangular plate is cumbersome and rigorous (Imrak & Gerdemeli, 2007). Bending analysis of rectangular plates is proposed using a combination of basic functions and finite difference energy techniques (Dey, 2001). The analysis of rectangular plate subjected to a uniformly distributed load with both ends fixed was done by Wojtaszak (1937) and Evans (1939). Evans (1937) developed and proposed design tables for deflections and moments for plate fixed on all edges and subjected to uniformly distributed load. Today, symbolic manipulation systems have become popular in engineering analysis (Betzer, 1990). The method is capable of manipulating both numbers and symbols. The computer-aided algebraic computations can considerably reduce tedious and rigorous analytical calculations and at the same time improve the results. In this research, it is shown how the symbolic
computer manipulation approach (Quick Basic) can be employed using the well-established variational method of Ritz to solve the bending problem of clamped isotropic rectangular plates with or without elastic foundations. The variational results obtained with this procedure will be compared with those of classical methods.

FORMULATION OF PLATE EQUATION
The general equation of plate is formulated using total potential Energy principles. Total potential Energy \( \Pi \) consists of strain energy of deformation \( U \) and potential Energy of external work \( W_e \), assuming the element of the structure under the transverse load remains elastic and is under adiabatic condition. Assuming Hooke's law is strictly obeyed, the Strain Energy of the plate is

\[
\mathcal{E} = \frac{1}{2} \sigma_y \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy} dtdt \quad \cdots (1)
\]

Where \( \sigma_x \) = normal stress along the x-axis
\( \sigma_y \) = normal stress along the y-axis
\( \tau_{xy} \) = shear stress along the x-y plane.
\( \varepsilon_x, \varepsilon_y \) and \( \gamma_{xy} \) are the respective strains on x, y, axes and x-y plane.

But
\[
\sigma_x = \frac{\varepsilon_x}{(1-\nu^2)} \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \quad \cdots (2a)
\]
\[
\sigma_y = \frac{\varepsilon_y}{(1-\nu^2)} \left( \frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \quad \cdots (2b)
\]
\[
\tau_{xy} = \frac{1-\nu}{(1-\nu^2)} \left( \frac{\partial^2 w}{\partial x \partial y} \right) \quad \cdots (2c)
\]

\[
\varepsilon_x = -\frac{\partial^2 w}{\partial x^2} \quad \cdots (3a)
\]
\[
\varepsilon_y = -\frac{\partial^2 w}{\partial y^2} \quad \cdots (3b)
\]
\[
\gamma_{xy} = -2 \frac{\partial^2 w}{\partial x \partial y} \quad \cdots (3c)
\]

Where \( E \) = modulus of Elasticity \( \nu \) = Poisson Ratio

The Strain Energy \( U \) can be written in terms of curvature by substituting the respective values of stresses and Strains of equations 2(a-c) and 3 (a-c) into equation (1) and simplifying to obtain

\[
U = \frac{1}{2} \frac{E \nu^2}{(1-\nu^2)} \iint \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{(\partial^2 w)}{\partial y^2} \frac{(\partial^2 w)}{\partial y^2} + 2\nu \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + 2(1-\nu) \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \right)| dxdy \quad \cdots (4)
\]

Integrating the first term of Equation (4) over the entire thickness of the surface from \(-h/2 \) to \( h/2 \) and simplifying, we obtain

\[
U = \frac{D}{2} \iint \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{(\partial^2 w)}{\partial y^2} \frac{(\partial^2 w)}{\partial y^2} + 2\nu \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + 2(1-\nu) \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \right) \right| dxdy \quad \cdots (5)
\]

Where \( D \) = Flexural Rigidity = \( \frac{E h^3}{12(1-\nu^2)} \)

In the present context, the plate is acted upon by uniformly distributed transverse load. Therefore, the external work \( W_e = \int q w(x,y) dxdy \). \cdots (6)

Therefore total potential Energy = \( U - W_e \). \cdots (7a)

\[
= \frac{D}{2} \iint \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{(\partial^2 w)}{\partial y^2} \frac{(\partial^2 w)}{\partial y^2} + 2\nu \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + 2(1-\nu) \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \right) - q w(x,y) dxdy \quad (7b)
\]

METHODOLOGY
The Direct variational method adopted here is Ritz which is based on minimum total potential Energy. Recall that the total potential Energy of plate from equation 7b is

\[
\Pi = \int D \left( \frac{\partial^2 w}{\partial x^2} \right)^2 + \frac{(\partial^2 w)}{\partial y^2} \frac{(\partial^2 w)}{\partial y^2} + 2\nu \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + 2(1-\nu) \frac{\partial^2 w}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} - q w(x,y) \frac{\partial^2 w}{\partial x \partial y} \quad (7b)
\]

Where \( W(x,y) \) is the plates deformation surface which is being approximated in this study as a variable – Separable polynomial as

\[
W(x,y) = \sum_{i=1}^{n} \sum_{j=1}^{m} \phi_i(x) \phi_j(y)
\]

Where \( \phi_i(x) \) is derivable from \( \phi_i(x) \) by replacing \( x \) by \( y \) and \( y \) by \( x \).

Equation (8) could be simplified further by putting

\[
\phi_i(x) = \phi_i(x), \phi_j(y) = \phi_j(y)
\]

Substituting equation (2) into equation (8), the deformation surface of the plate could now be written as

\[
W(x,y) = C_{11} h_1 + C_{12} h_2 + C_{21} h_3 + C_{22} h_4 + C_{11} h_5
\]

Where \( H = \begin{bmatrix} h_1 & h_2 & h_3 & \ldots & h_n \end{bmatrix} \), \( C = \begin{bmatrix} C_{11} & C_{12} & C_{21} & \ldots & C_{22} \end{bmatrix} \).

The functions of \( H \) polynomial of equation (10) must satisfy the kinematic boundary conditions and are linearly independent and continuous. These functions
of equation (10) are subsequently substituted into the total potential energy of equation (7b) above and on simplifying, becomes:

\[ \Pi = \int_\Omega \left[ \frac{1}{2} \left( A_{22} \frac{\partial^2 w}{\partial x^2} + A_{33} \frac{\partial^2 w}{\partial y^2} + 2A_{15} \frac{\partial^2 w}{\partial x \partial y} \right) \right] \, dx \, dy + \int_{\partial \Omega} q_{1} \, w \, ds - \int_{\partial \Omega} q_{2} \, w \, ds - \int_{\partial \Omega} q_{3} \, w \, ds \]

where \( A_{ij} = \int_{\Omega} H_{ij} \, dxdy \)

\[ \Pi = C_{1} \left( A_{1} + 2A_{2} + 2(1-\delta)A_{3} \right) [C] \]

For the equilibrium condition of the plate under the transverse loading to be maintained the total potential energy equation of (7b) above and on simplifying, becomes:

\[ \Pi = \int_\Omega \left[ \frac{1}{2} \left( A_{22} \frac{\partial^2 w}{\partial x^2} + A_{33} \frac{\partial^2 w}{\partial y^2} + 2A_{15} \frac{\partial^2 w}{\partial x \partial y} \right) \right] \, dx \, dy + \int_{\partial \Omega} q_{1} \, w \, ds - \int_{\partial \Omega} q_{2} \, w \, ds - \int_{\partial \Omega} q_{3} \, w \, ds \]

\[ \Pi = C_{1} \left( A_{1} + 2A_{2} + 2(1-\delta)A_{3} \right) [C] \]

where \( A_{1} = \int_{\Omega} H_{11} \, dxdy \), \( A_{2} = \int_{\Omega} H_{22} \, dxdy \), \( A_{3} = \int_{\Omega} H_{33} \, dxdy \), \( A_{4} = \int_{\Omega} H_{12} \, dxdy \), and

\[ B = \int_{\Omega} H_{11} \, dxdy \]

Subsequently the deflection at any point on the plate \((x, y)\) is determined by substituting the values for any arbitrary point on the plate into equation (17a). Moments at

**First Approximation (one-term of h polynomial)**

For this approximation, the deformation function representing the deflection surface of the plate is (using only the first term of equation (10))

\[ W(x, y) = C_{1}h_{1} \]  

\[ (16) \]

Where \( C_{1} = \) unknown coefficient

\[ h_{1} = \phi_{1}(x) \phi_{1}(y) \]

\[ (16a) \]

Where \( \phi_{1}(x) = \left( \frac{x}{a} \right)^{2} - 2 \left( \frac{x}{a} \right) \left( \frac{y}{b} \right) + \left( \frac{y}{b} \right)^{2} \]

\[ \phi_{1}(y) = \left( \frac{y}{b} \right)^{2} - 2 \left( \frac{x}{a} \right) \left( \frac{y}{b} \right) + \left( \frac{x}{a} \right)^{2} \]

\[ (16b) \]

Therefore, the deflection surface will be approximated by the equation:

\[ W(x, y) = C_{1} \left[ \left( \frac{x}{a} \right)^{2} - 2 \left( \frac{x}{a} \right) \left( \frac{y}{b} \right) + \left( \frac{y}{b} \right)^{2} \right] \]

\[ (17) \]

Assuming a square plate, \((a=b)\) the equation (17) is substituted into equation (7b), the results is integrated over the entire surface area \( A \) of plate via equation (12). The integrand is minimized and solved via equation 14, to obtain the unknown coefficient \( C_{1}=1.204 \). The determined coefficient is substituted into equation (16) to obtain the deformation surface of the plate in analytical form as:

\[ W(x, y) = C_{1} \left[ \left( \frac{x}{a} \right)^{2} - 2 \left( \frac{x}{a} \right) \left( \frac{y}{b} \right) + \left( \frac{y}{b} \right)^{2} \right] \]

\[ (17a) \]

The deflection at any point on the plate \((x, y)\) is determined by substituting the values for any arbitrary point \((x, y)\) into equation (17a). Moments at
similar point \((x, y)\) are obtained by putting equation (17a) into respective equations (15b-c).

Similarly, the results of the deflections and moments are determined by substituting the values of \((x, y)\) at any arbitrary point for various plate aspect ratios using equations (15a-d). The results are presented respectively on tables 1 and 2.

The accuracy of the results obtainable from this first approximation is poor although the response pattern of the deformation surface is good. For maximum deflection values displayed in table 1 for plate aspect ratios of \(1.0 \leq \left(\frac{c}{h/a}\right) \leq 2.0\), the errors range from nil for \(c = 1.0\) to about 11.82% for \(c = 2.0\). As would be expected, the deflection is evaluated with higher accuracy than the moment field \((M_x, M_y, M_{xy})\) which, in turn is evaluated to greater level of accuracy than the shear-force field \((Q_x, Q_y, q_x, q_y)\). This is because of the fact that the stress couples and the transverse Shear forces are proportional to the second and third derivatives of the displacement function respectively. Thus for plates having aspect ratio \(1.0 \leq c \leq 2.0\), the errors in maximum bending moment \((M_y)\) at \(a = b = \frac{h}{2}\) Table 2 range from 1.6% (at \(c = 1.6\)) to 20.08% at \((c = 1.1)\). For maximum bending moment along long span \((M_y)\), the errors range from 19.48% (at \(c = 1.0\)) to 58.25% (at \(c = 2.0\)). The edge (negative) moments have errors ranging from 0.5% (at \(c = 1.50\)) to 36.77% (at \(c = 2.0\)).

**Second Approximation (2-terms of \(b\) polynomial)**

In the second approximation, the deformation function describing the surface of the plate will be represented by 2 terms of \(b\) polynomial of equation (10).

\[
W(x, y) = Ca + Cb.
\]

where
\[
h_1 = \varphi_1(x), h_2 = \varphi_1(y).
\]

hence
\[
b_u = \frac{\varphi_1(x)}{\varphi_1(y)}; h_2 = \varphi_2(y).
\]

Therefore,
\[
W(x, y) = C\left[\frac{x}{a} - 2\frac{x}{a} + \frac{x}{a}\right] + C\left[\frac{y}{b} - 2\frac{y}{b} + \frac{y}{b}\right]
\]

\[
+C\left[\frac{x}{a} - 2\frac{x}{a} + \frac{x}{a}\right] + C\left[\frac{y}{b} - 2\frac{y}{b} + \frac{y}{b}\right] - (20a)
\]

Similarly, equation (20) is substituted into equation (7b), integrated via equation (11) and the integrand minimized and solved via equations (14) to obtain the coefficients \(C_1 = 5.527\), and \(C_2 = 1.340\). Therefore, in analytical form, the deflection surface of the plate for 2-term polynomial becomes,

\[
W(x, y) = 5.527\left\{\frac{x}{a} - 2\left(\frac{x}{a}\right)^2 + \left(\frac{x}{a}\right)^3\right\} + 1.340\left\{\frac{y}{b} - 2\left(\frac{y}{b}\right)^2 + \left(\frac{y}{b}\right)^3\right\} - (20a)
\]

The respective deflections at the centre of the plate and moments at the centre of span and edges are obtained for various plate aspect ratios by using equations (15a-d). The results are similarly presented on tables 1, 2, 3, 4, and 5. The two-term polynomial solution yields better accurate results than preceding one term polynomial. From table 1, it is observed that for plate aspect ratios \(1.0 \leq c \leq 2.0\), the errors in determined deflections range from 0.05% (at \(c = 1.0\)) to 4.5% (at \(c = 1.5\)). Evidently, the evaluated results of bending moments \((M_y, M_x)\) and transverse Shear forces corresponding to \(1.0 \leq c \leq 2.0\) (Table 2 and 3) are not as accurate as the displacement counterpart, although there is highly improve results over the corresponding one-term polynomial. As shown in tables 2 and 3, and for aspect ratios considered \((1.0 \leq c \leq 2.0)\) the errors of Short span moment \((M_y)\) range from nil (at \(c = 1.9\)) to 12.99% (at \(1.0\)) while that of long span moment range form 0.98% (at \(c = 1.5\)) to 9.49% (at \(c = 2.0\)). There is observed improvement in the accuracy of the results. The edge negative moments, errors range form 3.02% (c = 2.0) to 19.10% (c = 1.0) and Nil (c = 1.4) to 10.1% (at \(c = 2.0\)) for short span \((M_y)\) and long span \((M_x)\) respectively.

**Third Approximation (3-terms of \(b\) polynomial)**

The deformation surface of the plate for the third approximation will be represented by the 3-term polynomial of equation (10) as:

\[
W(x, y) = C_1 h_1 + C_2 h_2 + C_3 h_3.
\]

where
\[
h_1 = \varphi_1(x)\varphi_1(y), h_2 = \varphi_1(x), h_3 = \varphi_1(y).
\]

hence
\[
b_u = \frac{\varphi_1(x)}{\varphi_1(y)}; h_2 = \varphi_2(y).
\]

Therefore,
\[
W(x, y) = C_1\left[\frac{x}{a} - 2\left(\frac{x}{a}\right)^2 + \left(\frac{x}{a}\right)^3\right] + C_2\left[\frac{y}{b} - 2\left(\frac{y}{b}\right)^2 + \left(\frac{y}{b}\right)^3\right] + C_3\left[\frac{x}{a} - 2\left(\frac{x}{a}\right)^2 + \left(\frac{x}{a}\right)^3\right] - (20a)
\]

In the similar way of section (4.2), the unknown coefficients of \(C_1, C_2, C_3\) are evaluated to be 9.764, -1.326, and -1.326 respectively. The deformation surface of the plate in analytical form becomes:

\[
W(x, y) = 9.764\left\{\frac{x}{a} - 2\left(\frac{x}{a}\right)^2 + \left(\frac{x}{a}\right)^3\right\} - 1.326\left\{\frac{y}{b} - 2\left(\frac{y}{b}\right)^2 + \left(\frac{y}{b}\right)^3\right\} - 1.326\left\{\frac{x}{a} - 2\left(\frac{x}{a}\right)^2 + \left(\frac{x}{a}\right)^3\right\} - (20b)
\]

Similarly, the respective deflections and moments are evaluated and the results presented on tables 1, 2, 3, 4, and 5.
Higher accurate results are obtained with the 3 – term polynomial. For deflection, the maximum errors range from Nil (at c = 1.0, through c = 1.30) to 1.5% (at c = 2.0). The improvement in the accuracy of results in equally observed with the moment field (Mx, My). As depicted in table 2, the errors of moment range from 0.76% (at c = 1.10) to 4.36% (at c = 2.0). As evidence in table 3, the long span moments have errors ranging from 2.60% (c = 1.10) to 12.66% (c = 2.0). For edge moments (Mx, My), errors of short span solutions (Mx) range from 1.56% (c = 1.0) to maximum of 2.69% (c = 1.60). While long span (Mx) range from 0.15% (c = 1.30) to 10.33% (c = 2.0).

Fourth Approximation (four terms of h polynomials)

For the fourth approximation using four terms of h polynomials, the functional representing the deformation surface of the plate is written as:

$$W(x,y)=C_h+C_l+C_r+C_b,$$

where

$$h_n=\phi_n(x,y)=\phi_n(x)\phi_n(y)\quad \text{for } n=1,2,3,4.$$

Finally, the four terms polynomial of equation (22) is substituted into potential energy equation (10), integrated, minimized, and solved to obtain coefficients. Similar analyses are done for deflections and moments as in the proceeding sections to obtain deflections at centre and, mid span and edge moments. The results are shown on tables 1, 2 and 3. As indicated in the 3-term polynomial, the 4th Approximation solutions do not show marked difference from the 3rd thus indicating full convergence was obtained with 3-term h-polynomial for deflections, and all the moments’ fields $\left\{M_{xx},M_{yy},M_{xy},M_{yx}\right\}$ thus elucidating the validity and reliability of the variational procedure.

### Table 1-Maximum deflection coefficients ($\alpha$) of isotropic all round clamped rectangular plate under uniformly distributed load for various plate aspect ratios ($\delta=0.30$).

<table>
<thead>
<tr>
<th>Span ratio $(b/a)$</th>
<th>Classical method</th>
<th>Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 term of h</td>
<td>2 terms of h</td>
</tr>
<tr>
<td>1.0</td>
<td>0.00126</td>
<td>0.00130(0.05%)</td>
</tr>
<tr>
<td>1.1</td>
<td>0.00150</td>
<td>0.00154(2.67%)</td>
</tr>
<tr>
<td>1.2</td>
<td>0.00172</td>
<td>0.00175(1.74%)</td>
</tr>
<tr>
<td>1.3</td>
<td>0.00191</td>
<td>0.00194(1.57%)</td>
</tr>
<tr>
<td>1.4</td>
<td>0.00207</td>
<td>0.0020(0.97%)</td>
</tr>
<tr>
<td>1.5</td>
<td>0.00220</td>
<td>0.0022(0.45%)</td>
</tr>
<tr>
<td>1.6</td>
<td>0.00230</td>
<td>0.0023(0.43%)</td>
</tr>
<tr>
<td>1.7</td>
<td>0.00238</td>
<td>0.0023(0.82%)</td>
</tr>
<tr>
<td>1.8</td>
<td>0.00245</td>
<td>0.0024(0.41%)</td>
</tr>
<tr>
<td>1.9</td>
<td>0.00249</td>
<td>0.0024(0.40%)</td>
</tr>
<tr>
<td>2.0</td>
<td>0.00254</td>
<td>0.0025(1.18%)</td>
</tr>
</tbody>
</table>

### Table 2-Maximum short span moments coefficients ($\beta$) in all round clamped isotropic rectangular plates under uniform load for various plate aspect ratios ($\delta=0.30$).

<table>
<thead>
<tr>
<th>Span ratio $(b/a)$</th>
<th>Classical method</th>
<th>Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 term of h</td>
<td>2 terms of h</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0231</td>
<td>0.0275(19.05%)</td>
</tr>
<tr>
<td>1.1</td>
<td>0.0264</td>
<td>0.0317(20.08%)</td>
</tr>
<tr>
<td>1.2</td>
<td>0.0299</td>
<td>0.0352(17.72%)</td>
</tr>
<tr>
<td>1.3</td>
<td>0.0327</td>
<td>0.038(16.51%)</td>
</tr>
<tr>
<td>1.4</td>
<td>0.0349</td>
<td>0.0406(16.51%)</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0368</td>
<td>0.0427(16.33%)</td>
</tr>
<tr>
<td>1.6</td>
<td>0.0381</td>
<td>0.044(16.03%)</td>
</tr>
<tr>
<td>1.7</td>
<td>0.0392</td>
<td>0.0458(16.53%)</td>
</tr>
<tr>
<td>1.8</td>
<td>0.0401</td>
<td>0.0470(16.84%)</td>
</tr>
<tr>
<td>1.9</td>
<td>0.0407</td>
<td>0.0480(17.94%)</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0412</td>
<td>0.0488(18.45%)</td>
</tr>
</tbody>
</table>

The values in the bracket indicate the % deviation of the present study from the classical solution.

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Table 3-Maximum long span moments coefficients ($\beta$) in all round clamped rectangular plates under uniform load for various plate aspect ratios ($\alpha$=0.30).

<table>
<thead>
<tr>
<th>Span ratio = (b/a)</th>
<th>Classical method</th>
<th>Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 term of h</td>
<td>2 terms of h</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0231</td>
<td>0.0276(19.48%)</td>
</tr>
<tr>
<td>1.1</td>
<td>0.0231</td>
<td>0.0286(23.81%)</td>
</tr>
<tr>
<td>1.2</td>
<td>0.0228</td>
<td>0.0289(23.44%)</td>
</tr>
<tr>
<td>1.3</td>
<td>0.0222</td>
<td>0.0289(30.18%)</td>
</tr>
<tr>
<td>1.4</td>
<td>0.0212</td>
<td>0.0286(34.91%)</td>
</tr>
<tr>
<td>1.5</td>
<td>0.0203</td>
<td>0.0280(37.99%)</td>
</tr>
<tr>
<td>1.6</td>
<td>0.0193</td>
<td>0.0275(42.48%)</td>
</tr>
<tr>
<td>1.7</td>
<td>0.0182</td>
<td>0.0268(47.25%)</td>
</tr>
<tr>
<td>1.8</td>
<td>0.0174</td>
<td>0.0262(50.57%)</td>
</tr>
<tr>
<td>1.9</td>
<td>0.0165</td>
<td>0.0256(55.15%)</td>
</tr>
<tr>
<td>2.0</td>
<td>0.0158</td>
<td>0.0250(58.25%)</td>
</tr>
</tbody>
</table>

Table 4-Maximum short span edge moments coefficients ($\beta'$) in all round clamped isotropic rectangular plates under uniform load for various plate aspect ratios ($\alpha$=0.30).

<table>
<thead>
<tr>
<th>Span ratio = (b/a)</th>
<th>Classical method</th>
<th>Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 term of h</td>
<td>2 terms of h</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.0513</td>
<td>-0.0425(-17.15%)</td>
</tr>
<tr>
<td>1.1</td>
<td>-0.0581</td>
<td>-0.0507(-12.74%)</td>
</tr>
<tr>
<td>1.2</td>
<td>-0.0639</td>
<td>-0.0604(-36.77%)</td>
</tr>
<tr>
<td>1.3</td>
<td>-0.0687</td>
<td>-0.0648(-5.68%)</td>
</tr>
<tr>
<td>1.4</td>
<td>-0.0726</td>
<td>-0.0705(-2.89%)</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.0757</td>
<td>-0.0754(-0.40%)</td>
</tr>
<tr>
<td>1.6</td>
<td>-0.0780</td>
<td>-0.0795(-1.92%)</td>
</tr>
<tr>
<td>1.7</td>
<td>-0.0799</td>
<td>-0.0803(-3.88%)</td>
</tr>
<tr>
<td>1.8</td>
<td>-0.0812</td>
<td>-0.0805(-5.91%)</td>
</tr>
<tr>
<td>1.9</td>
<td>-0.0822</td>
<td>-0.0886(-7.78%)</td>
</tr>
<tr>
<td>2.0</td>
<td>-0.0829</td>
<td>-0.0907(-9.4%)</td>
</tr>
</tbody>
</table>

The values in the bracket indicate the % deviation of the present study from the classical solution.

Table 5-Maximum long span edge moments coefficients ($\beta'$) in all round clamped isotropic rectangular plates under uniform load for various plate aspect ratios ($\alpha$=0.30).

<table>
<thead>
<tr>
<th>Span ratio = (b/a)</th>
<th>Classical method</th>
<th>Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 term of h</td>
<td>2 terms of h</td>
</tr>
<tr>
<td>1.0</td>
<td>-0.0513</td>
<td>-0.0425(-17.5%)</td>
</tr>
<tr>
<td>1.1</td>
<td>-0.0538</td>
<td>-0.0419(-22.12%)</td>
</tr>
<tr>
<td>1.2</td>
<td>-0.0554</td>
<td>-0.0582(5.05%)</td>
</tr>
<tr>
<td>1.3</td>
<td>-0.0563</td>
<td>-0.0538(-31.97%)</td>
</tr>
<tr>
<td>1.4</td>
<td>-0.0568</td>
<td>-0.0360(-36.9%)</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.0570</td>
<td>-0.0335(-41.22%)</td>
</tr>
<tr>
<td>1.6</td>
<td>-0.0571</td>
<td>-0.0311(-45.31%)</td>
</tr>
<tr>
<td>1.7</td>
<td>-0.0571</td>
<td>-0.0287(-49.74%)</td>
</tr>
<tr>
<td>1.8</td>
<td>-0.0571</td>
<td>-0.0265(-53.90%)</td>
</tr>
<tr>
<td>1.9</td>
<td>-0.0571</td>
<td>-0.0245(-57.09%)</td>
</tr>
<tr>
<td>2.0</td>
<td>-0.0571</td>
<td>-0.0227(-60.24%)</td>
</tr>
</tbody>
</table>

The values in the bracket indicate the % deviation of the present study from the classical solution.
CONCLUSION
The well-known Ritz mathematical and variational method was developed and successfully applied in the analysis of uniformly loaded clamped isotropic rectangular plate. The tedious, time-consuming and rigorous computations have been circumvented using Quick Basic programming language.

The symbolic variational approach appears simple, acceptable and understandable by any Civil/Structural Engineer. The results obtained with the present study compare favourably with the classical solution (Timoshenko & Woinowsky-Kreer - 1959). In the course of this study, several methods of analyses especially the numerical methods were reviewed extensively. The most widely accepted classical method though acknowledge as satisfactory for most engineering problems are usually very difficult and rigorous. The modelled Ritz variational method formulated using total potential energy principle circumvents the rigorous procedure inherent in the classical solution through minimization principle.

The results of this procedure are obtained in analytical form thus, enabling the determination of deflection and moments at any arbitrary point of the plate unlike numerical methods that give results only at the nodal point. Also the results of moments at the clamped position compare favourably with the classical and numerical solution. This developed model can conveniently applied to thin rectangular of variable thickness and is therefore recommended for certain problems on plate analysis that do not have exact solution or their exact solutions are too complicated to be obtained analytically.

REFERENCES


