A Buried vertical Long Dip-Slip Fault in a Viscoelastic Half-Space Model of The Lithosphere

Subrata Kr. Debnath

Department of Basic Science and Humanities, Meghnad Saha Institute of Technology (A unit of Techno India Group), Nazirabad, P.O. Uchhepota, Via- Sonarpur, Kolkata-700150(INDIA),

Abstract
Most of the earthquake faults in North-East India, China, mid Atlantic-ridge, the Pacific seismic belt and Japan are found to be predominantly dip-slip in nature. In the present paper a buried long dip-slip fault is taken to be situated in a viscoelastic half space representing the upper lithospheric region of the Earth. A movement of the dip-slip nature across the fault occurs when the accumulated stress due to various tectonic reasons e.g. mantle convection etc, exceeds the local friction and cohesive forces across the fault. The movement is assumed to be slipping in nature; expressions for displacements, stresses and strains are obtained by suitable mathematical methods. A suitable numerical technique has been adopted for computer simulation. A detailed study of these expressions may give some ideas about the nature of stress accumulation in the system, which in turn will be helpful in formulating an earthquake prediction programme.

Keywords: aseismic period, buried dip-slip fault, earthquake prediction, mantle convection, plate movements, stress accumulation, tectonic process, viscoelastic half space

INTRODUCTION
Modelling of dynamic processes leading to an earthquake is one of the main concerns of seismologists and geotechnical engineers. Two consecutive seismic events (Two consecutive earthquakes) in a seismically active region are usually separated by a long aseismic period during which slow and continuous aseismic surface movements are observed with the help of sophisticated measuring instruments like strain meter and tilt meter etc. Such aseismic surface movements indicate that slow aseismic change of stress and strain are occurring in the region which may eventually lead to sudden or creeping movements across the seismic faults (may be strike-slip, finite or long or dip-slip or both) situated in the region.

It is therefore seems to be an essential feature to identify the nature of stress and strain accumulation/release in the vicinity of seismic faults situated in the region by studying the observed ground deformation during the aseismic period. A proper understanding of the mechanism of such aseismic quasi static deformation may give us some precursory information regarding the impending earthquake.

It is observational fact that while some faults are strike slip (finite or long) in nature, there are faults (e.g.: Sierra Nevada/Owens valley: Basin and Range faults, Rocky Mountains, Himalayas, Atalanti fault of central Greece-a steeply dipping fault with dip 60, 80(deg)) where the surface level changes during the motion i.e. the faults are dip-slip in nature.


In most of these works the medium were taken to be elastic and /or viscoelastic, some authors preferred layered model with elastic layer(s) over elastic or viscoelastic half space. In the present case we consider a long buried dip-slip fault situated in a viscoelastic half space. The medium is taken to be under the influence of tectonic forces due to mantle convection or some related phenomena. The fault is assumed to undergo a slipping movement when the stresses in the region exceed certain threshold values. 

In our paper, we consider a viscoelastic half space to represent the upper part of the lithosphere-asthenosphere system, with constant rigidity $(2.0 \times 10^8$ Mpa) and viscosity $(10^{20} - 10^{21}$ Pa s) following the observational data mentioned by Chift, P. Lin, J. Barcktiausen, U. (2002), Karato Shun-Ichiro, (2010).

It may be stated that a thin elastic layer overlying an elastic/ viscoelastic half space is likely to be a more preferable model for the system. But, numerical computational works indicate that the presence of a layer does not lead to any significant qualitative changes in the nature of the stress and strain accumulation in the model, only a small 10 percent quantitative change were observed. Analytical expressions for displacements stresses and strains are obtained both before and after the fault movement using appropriate mathematical technique involving integral transformation, Green’s function. Numerical computational works have been carried out with suitable values of the model parameters and the nature of the stress and strain accumulation in the medium have been investigated.

**Formulation**

We consider a long dip-slip fault $F$, width D situated in a viscoelastic half space of linear Maxwell type. A Cartesian coordinate system is used with a suitable point O on the strike of the fault as the origin, the strike of the fault along the $Y_1$ axis and $Y_2$ axis is as shown in Figure 1, and $Y_3$ axis pointing downwards. 

We choose another coordinate system $Y'_1$, $Y'_2$ and $Y'_3$ axes as shown in Figure 1 below, so that the fault is given by $F$: $(y'_1 = 0, 0 \leq y'_2 \leq D)$. Let 0 be the dip of the fault $F$. Let $\nu_y$, $\omega$ be the displacement component along the $Y_2,Y_3$ axes and $\tau_{22}, \tau_{23}, \tau_{33}$ be the stress components and $e_{22}, e_{23}$ and $e_{33}$ be the strain components respectively.

![Figure 1. Section of the model by the plane $y_2=0$.](image)

For a viscoelastic Maxwell type medium the constitutive equations are taken as:

$$\begin{align}
\frac{1}{\eta} + \frac{1}{\mu} \frac{\partial}{\partial t} \tau_{22} &= \frac{\partial}{\partial t} (e_{22}) = \frac{\partial}{\partial t} \left( \frac{\partial \nu}{\partial y_2} \right) \\
\frac{1}{\eta} + \frac{1}{\mu} \frac{\partial}{\partial t} \tau_{23} &= \frac{\partial}{\partial t} (e_{23}) = \frac{1}{2} \left( \frac{\partial \nu}{\partial y_3} + \frac{\partial \omega}{\partial y_2} \right) \\
\frac{1}{\eta} + \frac{1}{\mu} \frac{\partial}{\partial t} \tau_{33} &= \frac{\partial}{\partial t} (e_{33}) = \frac{\partial}{\partial t} \left( \frac{\partial \omega}{\partial y_3} \right)
\end{align}$$

(1.1) (1.2) (1.3)

where $\eta$ is the effective viscosity and $\mu$ is the effective rigidity of the material.

**The stresses satisfy the following equations:**

(assuming quasistatic deformation for which the inertia terms are neglected).

$$\frac{\partial}{\partial y_2} (\tau_{22}) + \frac{\partial}{\partial y_3} (\tau_{23}) = 0$$

(1.4) 

$$\frac{\partial}{\partial y_2} (\tau_{32}) + \frac{\partial}{\partial y_3} (\tau_{33}) = 0$$

(1.5)

Where $(-\infty < y_2 < \infty), y_3 \geq 0, t \geq 0$ (Assuming the body forces do not change during the fault movement).

**The boundary conditions are taken as, with $t=0$ representing an instant when the medium is in aseismic state:**

$$\tau_{22} (y_2, y_3, t) = \tau_\infty (t) \cos \theta$$

$$| y_2 | \rightarrow \infty, y_3 \geq 0, t > 0$$

(1.6)

On the free surface

$$y_3 = 0 ( -\infty < y_2 < \infty, t \geq 0)$$

$$\tau_{23} (y_2, y_3, t) = 0$$

(1.7) 

$$\tau_{33} (y_2, y_3, t) = 0$$

(1.8)

Also, as $y_3 \rightarrow \infty ( -\infty < y_2 < \infty, t \geq 0)$

$$\tau_{23} (y_2, y_3, t) = 0$$

(1.9)
where \( \tau_s(t) \) is the shear stress maintained by mantle convection and other tectonic phenomena far away from the fault.

### The Initial Conditions are

Let \((v_{0x}(0), v_{0y}(0), e_{0x})\) and \((e_{0y})\) at \(t = 0\) which are functions of \(y_2, y_3\) and satisfy the relations (1.1)-(1.10).

(B) **Solutions in the absence of any fault dislocation:** The boundary value problem given by (1.1)-(1.10), can be solved (as shown in the Appendix-1) by taking Laplace transform with respect to time ‘\(t\)’ of all the constitutive equations and the boundary conditions. On taking the inverse Laplace transform we get the solutions for displacements, stresses as:

\[
v(y_2, y_3, t) = (v_0) + y_2 \cos \theta / \mu \times \int_0^\infty \tau_s(\tau) d\tau
\]

\[
w(y_2, y_3, t) = (w_0) + (y_3 / \mu) \times \int_0^\infty \tau_s(\tau) d\tau \sin \theta
\]

\[
\tau_{22} = \tau_{22} (0) \cos \theta - \tau_{22} (0) e^{-\mu \eta \tau} [\sin \theta]
\]

\[
\tau_{23} = (\tau_{23})_0 e^{-\mu \eta \tau}
\]

\[
\tau_{33} = [\tau_{33} (t) - \tau_{33} (0)] e^{-\mu \eta \tau} \sin \theta
\]

From the above solution we find that \(\tau_{22}\) increases with time and tends to \(\tau_s(0) \cos \theta\) as \(t\) tends to \(\infty\), while \(\tau_{23}\) tends to zero, but \(\tau_{33}\) tends to \(\tau_{33}(0) \sin \theta\). We assume that the geological conditions as well as the characteristic of the fault is such that when the stress component \(\tau_{23}\) reaches some critical value, say \(\tau_c < \tau_s(0) \cos \theta\) the fault \(F\) starts slipping.

For bounded stress and strains, the function should satisfy the following conditions as discussed in [21].

(C1) Its value will be maximum on the free surface.

(C2) The magnitude of the creep will decrease with \(y_3\) as we move downwards and ultimately tends to zero near the lower edge of the fault.

\((y_2') = 0, y_3' = D\)

The function \(g(x')\) should satisfy the above conditions.(We call it creep function)

(B) **Solutions after the fault movements:** We assume that after a time \(T_f\), the stress component \(\tau_{23}\) (which is the main driving force for the dip-slip motion of the fault) exceeds the critical value \(\tau_c\) and the fault \(F\) starts slipping, characterized by a dislocation across the fault given by (Appendix-2).
\[ A_3 = (x_3^2 - 2x_3'(y_2 \cos \theta + y_3 \sin \theta) + y_2^2 + y_3^2) \]
\[ (-\cos \theta) - (y_2 \sin \theta - y_3 \cos \theta) \times (2y_3 - 2x_3' \cos \theta) \]
\[ B_3 = x_3^2 - 2x_3'(y_3 \cos \theta + y_3 \sin \theta) + y_2^2 + y_3^2 \]
\[ C_3 = (x_3^2 - 2x_3'(y_2 \cos \theta - y_3 \sin \theta) + y_2^2 + y_3^2) \times \cos \theta - (y_2 \sin \theta + y_3 \cos \theta) \times (2y_3 - 2x_3' \sin \theta) \]
\[ D_3 = x_3^2 - 2x_3'(y_3 \cos \theta - y_3 \sin \theta) + y_2^2 + y_3^2 \]
\[ e_{23}(y_2, y_3, \theta, t) = \left( \frac{1}{2} \right) (e_{23})_0 + H(t - T_1) / (2\pi) \]
\[ W \phi_3(y_2, y_3, \theta, t) \]
\[ y_3 \text{ (in km.)} \]
\[ \text{Figure 2: Variation of the vertical component of surface displacement } w \text{ with } y_2 \text{ for } y_2 = 0, t_1 = 1 \text{ year, } \theta = 90 \text{ (in deg).} \]
\[ y_3 \text{ (in km.)} \]
\[ \text{Figure 3: Variation of the vertical component of surface displacement } w \text{ with } y_3 \text{ for } y_3 = 5 \text{ km, } t_1 = 1 \text{ year, } \theta = 60 \text{ (in deg).} \]
(B) Variation of the main driving stress $t_{x'y'}$ with depth due to the movement across $F$.

Figures (4) – (7) show the variation of $t_{x'y'}$ with depth $y_3$ for various $\theta$ and some specific values of $y_2$.

In Figure 4, it is found that for a dip-slip fault $\theta=30$ (in degree) and at a point very near to the fault $y_2=9$ km, $t_{x'y'}$ undergoes sudden change (in one year) by an amount $=0.5$ bar, due to the slipping movement across $F$ at a depth of about $y_3=10$ km. Thereafter, the stress accumulation decreases and become zero at a depth of about $y_3=50$ km. Then starts releasing up to a depth of about $y_3=50$ km. and then become zero for $y_3>50$ km.

$y_3$ (in km.)

Figure 4 Variation of the stress component $t_{x'y'}$ with $y_3$ for $y_2=9$ km, $t_f=1$ year, $\theta=30$ (in deg) due to the fault movement.

$y_3$ (in km.)

Figure 5 Variation of the stress component $t_{x'y'}$ with $y_3$ for $y_2=9$ km, $t_f=1$ year, $\theta=60$ (in deg) due to the fault movement.

Figure 5 shows the variation of stress component $t_{x'y'}$ for $y_2=9$ km, $t_f=1$ year and dip angle $\theta=60$ (degree) with the depth due to the slip across the fault.

It is observed that the stress initially releases up to a depth of about $y_3=2$ km, then starts accumulating with a higher rate up to a depth of about 10 km. attaining a maximum of about 0.025 bar in one year, and then decreases rapidly up to a depth of about 75 km, then with a slower rate and gradually tends to zero at a depth of about 100 km.

$y_3$ (in km.)

Figure 6 Variation of the stress component $t_{x'y'}$ with $y_3$ for $y_2=10$ km, $t_f=1$ year, $\theta=90$ (in deg) due to the fault movement.

We see rate of stress accumulation decreases in the region $0 \leq y_3 \leq 10$ km then it increases up to 50 km. attaining a maximum there of magnitude of 0.024 bar in one year and becomes 0 at a depth about 300 km. as the fig 6 shows. Thus in the above discussion we see that due to slip movement in the dip-slip fault, there are regions where stress get released and there are certain other regions where stress accumulates. The rate of stress release/accumulation depends essentially on the dip-angle $\theta$ and the distance $y_2$ from the fault.

(C) Prediction of the next event:

Let $\tau_c$ be the critical stress of the region of consideration and $t_c$ be the critical time, then we have,

$$t_c = (\eta / \mu) \log \left[ (\tau_{c3} / \tau_c) \right] \quad (2.6)$$

a numerical calculations show that $t_c=77$ years(app.). Thus the first slip occurs after about 77 years. Let due to the slip across the fault 80 percent of the total accumulated stress is dropped. Thus 20 percent of the total accumulated stress is now acts as the new initial stress.

Therefore, $(\tau_{c3})_{0,new} = 6 \times 10^7$ dynes/cm$^2$.

$\tau_{33}(y_2, y_3, \theta, t) = (\tau_{c3})_{0,new} \times e(\mu / \eta) t$

$+ [H (t - T_f) / (2 \pi)] e(-\mu / \eta) t$

$\int_D g(x'_i)[A_1 / B_2 + C_2 / D_2] dx'_i$

is the new stress equation.

If we let $t_c = (t_c)_o$ be the $n^{th}$ time to reach the critical stress $	au_c$, it will be given by,
where $A_1$ and $B_1$ are arbitrary constant which are independent of $y_2$ and $y_3$, to be determined using the initial and boundary conditions as above.

Using the boundary and the initial conditions we get,

$$A_1 = \frac{1}{p} \left[ 1 + \frac{P}{\eta} \right] \left( \tau_\alpha(0) - \tau_\alpha(0) \right)$$

and, $B_1 = 0$.

On taking inverse Laplace transformation we get,

$$v(y_2, y_3, t) = (v)_0 + \left( y_2 \sin \theta / \mu \right) \int_0^{\tau_\alpha(\tau) dt}$$

Similarly we can get the other components of the displacements.

$$w(y_2, y_3, \theta, t) = (w)_0 + \left( y_2 \sin \theta / \mu \right) \int_0^{\tau_\alpha(\tau) dt}$$

The stresses are given by,

$$\tau_{23} = (\tau_{23})_0 \cos \theta - \left[ \tau_{23}(0) \cos \theta - (\tau_{23})_0 \right] e^{-\mu \eta \tau}$$

$$\tau_{31} = (\tau_{31})_0 e^{-\mu \eta \tau}$$

Using the displacements the strains can also be found out to be,

$$e_{22}(y_2, y_3, \theta, t) = (e_{22})_0$$

$$e_{33}(y_2, y_3, \theta, t) = \left[ \frac{1}{2} \right] (e_{33})_0$$

Appendix

Solutions after the fault movement

We assume that after a time $T_1$ the stress component $\tau_{23}$ (which is the main driving force for the dip-slip motion of the fault) exceeds the critical value $\tau_0$ and the fault $F$ starts slipping. Then we have an additional condition characterizing the dislocation in $w$ due to the slipping movement as:

$$\left[ (w) \right]_F = Wg(y_1)H(t_1)$$

(4.1)

where, $H(t_1)$ is the Heaviside function and $[(w)F]$. The discontinuity of $w$ across $F$ is given by

$$\left[ (w) \right]_F = \lim_{(y_2) \rightarrow 0} (w) - \lim_{(y_2) \rightarrow 0} (w)$$

(4.2)

$$y_2 = 0 \leq y_2' \leq D$$

Taking Laplace transformation in (4.1) we get,

$$[(w)]_F = Wg(y_1')$$

(4.3)

The fault creep commences across $F$ after time $T_1$, clearly,

$$[(w)]_F = 0$$

for $t_1 \leq 0$, where $t_1 = t - T_1$, $F$ is located in the region $(y_2' = 0, 0 < y_2' < D)$.

We try to find the solution as,

$$v = (v)_h + (v)_l, w = (w)_h + (w)_l$$

where $(v)_l$ and $(v)_h$, $(w)_l$ and $(w)_h$ are the solution across the fault.
\( \tau_{zz} = (\tau_{zz} h + (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) + (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) + (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) + (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) + (\tau_{zz} l) \) (4.4)

where \((v_1), (w_1), (\tau_0),\) are continuous throughout the model and are given by (A). While the second part \((v_2), (\tau_2),\) are obtained by solving modified boundary value problem as stated below. We note that \((v_2)\) is continuous even after the fault slip, so that \([v_2] = 0\), while \((w_2)\) satisfies the dislocation condition given by (4.2).

The resulting boundary value problem can now be stated as:\((w_2)\), satisfies 2D Laplace equation as,

\[ \nabla^2 (w) = 0 \]

(4.5)

where, \((w)\) is the Laplace transformation of \((w_2)\), with the modified boundary condition,

\[ \tau_{zz} = (\tau_{zz} h + (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) + (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) + (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) + (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) + (\tau_{zz} l) \] as \(y_2 \rightarrow \infty, y_3 \rightarrow 0\) \((1.10b)\)

\[ \tau_{zz} = (\tau_{zz} h - (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) - (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) - (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) - (\tau_{zz} l), \tau_{zz} = (\tau_{zz} h) - (\tau_{zz} l) \] as \(y_2 \rightarrow (0, \infty)\) \((1.10b)\)

and the other boundary conditions are same as before.

We solve the above boundary value problem by modified Green’s function method following Maruyama (1966), Rybicki(1971) and the correspondence principle.

Let \(Q(y_2, y_3)\) be any point in the field and \(P(x_2, x_3)\) be any point in the fault, then we have,

\[ (w) = \int_{F} W(g(x'))G(x', y') dx' \]

(4.6)

where,

\[ G(x', y') = \frac{1}{2\pi} \left[ \frac{y_2 - x_2}{(y_2 - x_2)^2 + (y_3 - x_3)^2} \right] \]

(4.7)

\[ G(x', y') = \frac{1}{2\pi} \left[ \frac{y_2 - x_2}{(y_2 - x_2)^2 + (y_3 - x_3)^2} \right] \]

(4.8)

\[ (w) = \int_{F} W(g(x')) \]

(4.9)

Where,

\[ \phi_1(y_2, y_3, \theta) = \int_{F} g(x') \]

(4.10)

Taking inverse Laplace transformation,

\[ (w) = W\phi_1(y_2, y_3, \theta)H(t_1) \]

where, \(H(t_1)\) is the Heaviside step function, which gives the displacement at any points \(Q(y_2, y_3)\).

We also have,

\[ (\tau_{zz}) = 0 \]

(4.10)

\[ \phi_1(y_2, y_3, \theta) = \frac{p}{\mu \eta} \phi_1(y_2, y_3, \theta) \]

(4.10)

\[ \frac{\partial}{\partial y_3} \phi_1(y_2, y_3, \theta) \]

LIMITATION OF THIS RESEARCH

The prediction of the next earthquake is done from the given model parameters which have significant role in this work, being a theoretical researcher I have to be.

REFERENCES


Matsu'ura, M. Sato, R. 1975. Static deformation due to fault spreading over several layers in multi-layered medium part-I-Strain and tilt; jour. of the phys. of the Earth, vol. 23, no.1,12-33.


Okada, Y. 1992. Internal deformation due to shear and tensile faults in a half-space, B.S.S.A. vol.82, No.2. 1018-1040.


